Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3 **Branch: B. Tech (All)**

Semester : 1 **Date : 28/11/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) n^{th} derivative of $y=\sin^3 x$ is

(A) $\frac{3}{4}\sin\left(x+\frac{n\pi}{2}\right)-\frac{3^n}{4}\sin\left(3x+\frac{n\pi}{2}\right)$ (B) $\frac{3^n}{4}\sin\left(3x+\frac{n\pi}{2}\right)-\frac{3}{4}\sin\left(x+\frac{n\pi}{2}\right)$

(C) $\frac{3}{4}\sin\left(3x+\frac{n\pi}{2}\right)-\frac{3^n}{4}\sin\left(x+\frac{n\pi}{2}\right)$ (D) none of these

b) If $y=\frac{1}{x}$ then y_n equal to

(A) $\frac{(-1)^n n!}{x^n}$ (B) $\frac{(-1)^n n!}{x^{n+1}}$ (C) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ (D) none of these

c) If $y=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+.....$ then x equal to

(A) $y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+.....$ (B) $y-\frac{y^3}{3}+\frac{y^5}{5}-.....$ (C) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+.....$

(D) none of these

d) The series $1+\frac{x^2}{2!}+\frac{x^4}{4!}+.....$ represent expansion of

(A) $\sin x$ (B) $\log(1+x)$ (C) $\cos x$ (D) $\cosh x$

e) $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}=$ _____

(A) $\log ab$ (B) $\sqrt{ab}$ (C) $\frac{1}{2}\log ab$ (D) none of these

f) $\lim_{x\to\infty}\frac{x^n}{e^{ax}}=$ _____

(A) – 1 (B) 0 (C) 1 (D) none of these

g) If $u = y^x$, then $\frac{\partial u}{\partial y}$ is

(A) xy^{x-1} (B) $y^x \log y$ (C) 0 (D) none of these

h) If $u = f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ (C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$ (D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

i) If $u(x, y, z) = 0$ then the value of $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) – 1 (C) 0 (D) none of these

j) If $x = r\cos\theta,\ y = r\sin\theta,$ then $\frac{\partial(r, \theta)}{\partial(x, y)}$ is equal to

(A) 1 (B) r (C) $1/r$ (D) 0

k) The value of $\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$ is

(A) – 1 (B) 0 (C) – i (D) i

l) If 1, $\alpha_1,\ \alpha_2,\ .....,\ \alpha_{n-1}$ are the n roots of unity, then $(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1})$ is equal to

(A) $n-1$ (B) n (C) – 1 (D) none of these

m) A square matrix A is called orthogonal if

(A) $AA^{-1} = I$ (B) $A^2 = A$ (C) $A^T = A^{-1}$ (D) $A^2 = I$

n) If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & x & 4 \\ 1 & -1 & 1 \end{bmatrix}$ is 3, then x is not equal to

(A) 3 (B) 4 (C) 5 (D) 6

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n. **(5)**

b) Expand $\tan^{-1} x$ up to the first four terms by Maclaurin's series. **(5)**

c) If $V = \frac{1}{r}$ where $r^2 = x^2 + y^2 + z^2$ then show that $V(x,\ y,\ z)$ satisfies Laplace's equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. **(5)**

b) Prove that $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} \cdots\cdots$ **(5)**

c) Evaluate: $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)$ **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **(5)**

b) If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$. **(5)**

c) Expand $\log x$ in powers of $(x-2)$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u=\sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then prove that **(5)**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=-\cot u\left(\cot^2 u+2\right).$$

b) Evaluate: $\lim_{x\to 0}\left(\frac{a}{x}-\cot\frac{x}{a}\right)$ **(5)**

c) If $y=\cos x\cos 2x\cos 3x$ then find y_n. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Find the approximate value of $\sqrt{27}\sqrt[3]{1021}$ using partial differentiation. **(5)**

b) If $x_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$ then prove that $\lim_{n\to\infty}x_1x_2x_3\ldots x_n=-1$. **(5)**

c) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}1 & -2\\ -5 & 4\end{bmatrix}$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Reduce the matrix $A=\begin{bmatrix}2 & 3 & 4 & 5\\ 3 & 4 & 5 & 6\\ 4 & 5 & 6 & 7\\ 9 & 10 & 11 & 12\end{bmatrix}$ to the normal form and find its rank. **(5)**

b) Expand $\sin^5\theta\cos^2\theta$ in a series of sines of multiples of θ. **(5)**

c) Prove that $\sec h^{-1}(\sin\theta)=\log\cot\frac{\theta}{2}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Examine for linear dependence of vectors $(1,\ 2,\ -1,\ 0)$, $(1,\ 3,\ 1,\ 2)$, $(4,\ 2,\ 1,\ 0)$, $(6,\ 1,\ 0,\ 1)$ and find a relation between them if dependent. **(5)**

b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) Examine whether the following equations are consistent and solve them if they are consistent. **(4)**

$2x+6y+11=0,\ 6x+20y-6z+3=0,\ 6y-18z+1=0$